

## Experimental Vibration Investigation of Annular Plates

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Free vibration,  
Annular plate,  
Modal analysis,  
Numerical method.

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### Abstract

In the present research, the free vibrational behavior of annular plates is investigated. The plate is considered to be made up of ST37 steel. An experimental modal analysis is performed on a free-free annular plate using the ModalVIEW software to obtain the natural frequencies of vibration and corresponding mode shapes. A three-dimensional finite element model is also constructed using Abaqus 6.10 software. The natural frequencies and corresponding mode shapes meet an excellent agreement in comparison with those obtained via the numerical analysis. This study provides guidance on modal analysis and vibration measurement of annular plates.

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### 1. Introduction

Circular plates with holes are extensively used in engineering structures, e.g. missiles, aircraft, etc., in order to reduce the weight of the whole structure, to increase the range of inspection or to satisfy other engineering applications. The existence of the hole in a circular plate can significantly affect the natural frequencies and mode shapes of the structure. Therefore, studying the vibrational behavior of circular plate with a central hole, i.e., annular plate is of high importance. Till now, several research works have been conducted on the vibration analysis of these structures based on analytical, numerical and experimental methods, such as the energy approach, the mode subtraction approach, finite element method and etc. [1-8]. Based on numerical and experimental approaches, Cheng et al. [9] performed a detailed study on the vibration characteristics of annular plates. The effects of the eccentricity, hole size and boundary condition on vibration modes are investigated. They also carried out experimental modal analysis on a clamped-free annular plate and arrived at the results in good agreement with those obtained by numerical analysis.

In recent years, the vibration analysis of laminated composite and functionally graded annular plates has been a topic of practical interest and attracted much attention. Nallim and Grossi [10] performed the free transverse vibration analysis of symmetrically laminated solid and annular

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elliptic and circular plates using Rayleigh-Ritz method. Ovesy and Fazilati [11] applied the finite strip method based upon a Reddy type, third order shear deformation theory for investigating the buckling and free vibrational behavior of thick plates containing internal cutouts.

The buckling behavior of laminated composite circular plates having circular holes and subjected to uniform radial load was investigated using the finite element method by Baltaci et al. [12]. They also studied the influences of changes in the hole size, location of the hole, thickness and boundary conditions on the buckling load. Based on the three-dimensional theory of elasticity and a combination of state-space method and DQM, Nie and Zhong [13] used a semi-analytical approach for obtaining the vibration frequencies and dynamic response of functionally graded circular plates. Seifi et al. [14] studied the buckling behavior of composite annular plates under uniform internal and external radial edge loads have been investigated using energy method. Jodaei et al. [15] used a state-space based DQM to analyze the free vibration behavior of functionally graded annular plates. They also modeled the plate by artificial neural network for different boundary conditions. Further, the influence of thickness of the annular plate, material property graded index and circumferential wave number on the non-dimensional natural frequencies of the annular plates with different boundary conditions were investigated. Very recently, Rahimi et al. [16] proposed a three-dimensional elasticity theory by means of a state-space based differential quadrature method for free vibration analysis of fiber metal laminate annular plate. The kinds of composite material and metal layers were considered to be S2-glass and aluminum, respectively. A comparison also made with the numerical results reported by Abaqus 6.10 software which showed an excellent agreement.

The aim of the present research is to investigate the free vibrational behavior of annular plates experimentally and numerically. In section 2, experimental modal analysis is performed on a free-free annular plate. The plate is considered to be made up of ST37 steel. ModalVIEW software is implemented for acquiring the vibration signals and obtaining the frequency response functions (FRFs). In section 3, a three-dimensional finite element model is constructed using Abaqus 6.10 software and the natural frequencies of vibration and corresponding mode shapes are obtained. The results show an excellent agreement with those achieved via experimental setup. Moreover, the effect of aspect ratio of annular plates on the vibration frequencies of these structures are examined.

## **2. Experimental Modal Analysis**

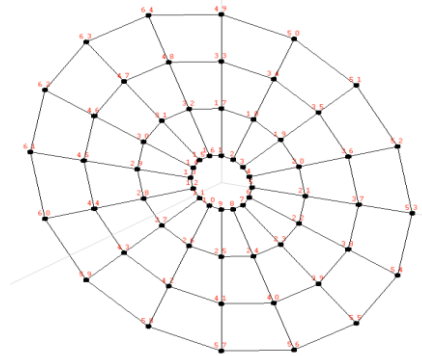
Modal analysis is the identification of vibration characteristics of elastic structures. It consists of describing a system by its modal parameters, i.e., natural frequencies, natural damping and natural modes. For the present analysis, National Instrument (NI) analyzer and ModalVIEW software are

implemented for acquiring the vibration signals and processing them. This software performs modal analysis using experimental technique by curve fitting the FRFs from modal testing experiment .

In general, performing modal analysis of a structure involves the following steps:

- Building a structure
- FRF Measurement
- Estimating modal parameters
- Animating mode shapes of structure

Figure 1 depicts the structure of an annular plate with 64 nodes. The effective diameter of the steel plates is 250 mm, with a thickness of 2 mm, and the diameter of the hole is 40 mm. In order to simulate the free-free boundary condition, the annular plate is suspended from the ceiling using one flexible string. The setup for the present experimental modal analysis is shown in Figure 2. After creating the structure of the annular plate, measurement direction should be defined for each measured node.



**Figure 1.** Structure of an annular plate built by ModalVIEW structure model drawing

FRF is used as data source for estimating the modal parameters of a structure. To obtain the FRF of the annular plate structure, the vibration response and force excitation from different points and directions on the plate are simultaneously measured by data acquisition hardware. For the present under test structure, a roving hammer impact test is used. The accelerometer is fixed at point 47. In order to generate excitation signal, a PCB hammer with sensitivity of 2.25 mV/N is used and the vibration amplitude at the measuring locations are sensed by a PCB accelerometer with sensitivity of 3.30 mV/N. FRF measurements are obtained by impacting at different points in the direction normal to the plate structure by using an impact hammer. Once a set of FRFs measured between a pair of degree of freedoms (DOFs) on the plate structure, modal parameters can be estimated by curve fitting FRFs in a specified frequency range. Each peak in FRF measurements indicates at least one mode.



**Figure 2.** Setup for the experimental modal analysis

### 3. Finite Element Modeling

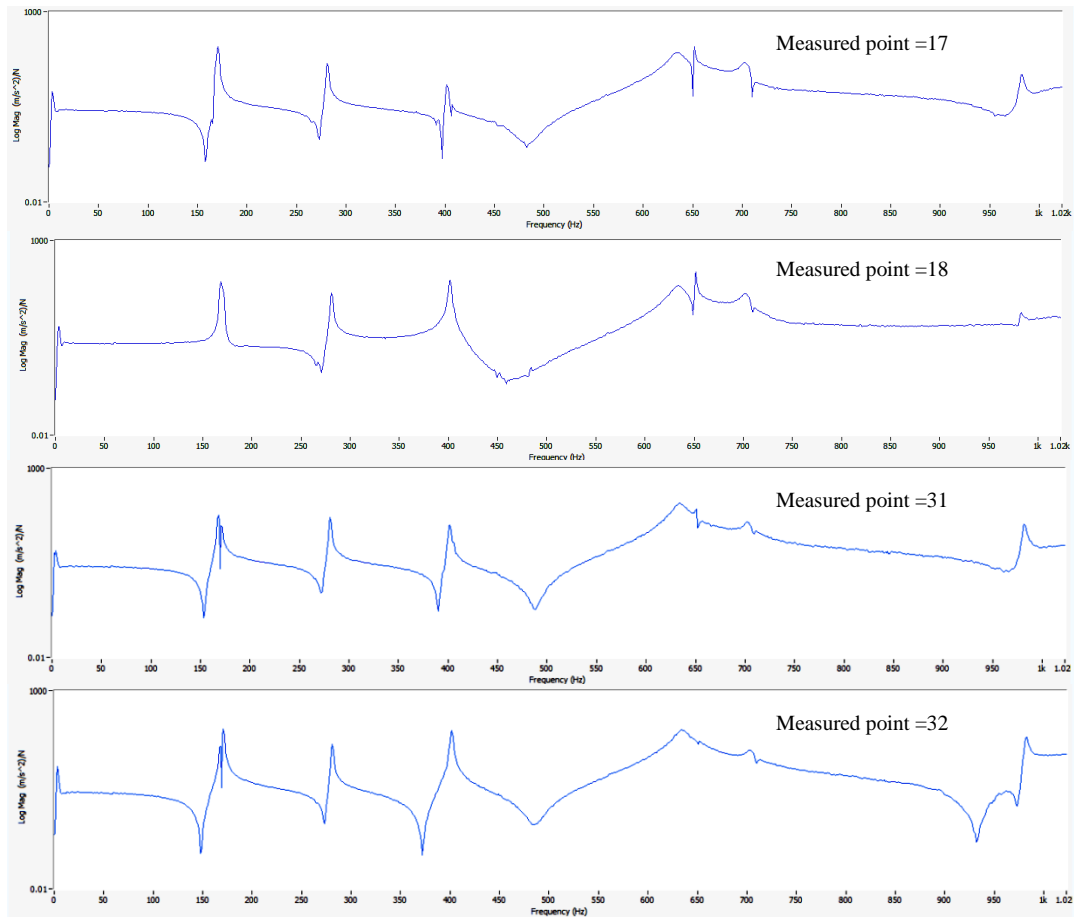
A three-dimensional finite element model of the annular plate is constructed in Abaqus 6.10 software. Following are the material properties for the specimen plates. Young's modulus  $E = 200 \text{ GPa}$ , Poisson's ratio  $\nu = 0.3$  and Density of the material  $\rho = 7850 \text{ kg/m}^3$ . The plate is meshed by shell-type elements. Using the free vibration analysis in Abaqus, the natural frequencies and associated mode shapes are achieved and will be then compared with those given by the experimental modal analysis.

### 4. Results and Discussions

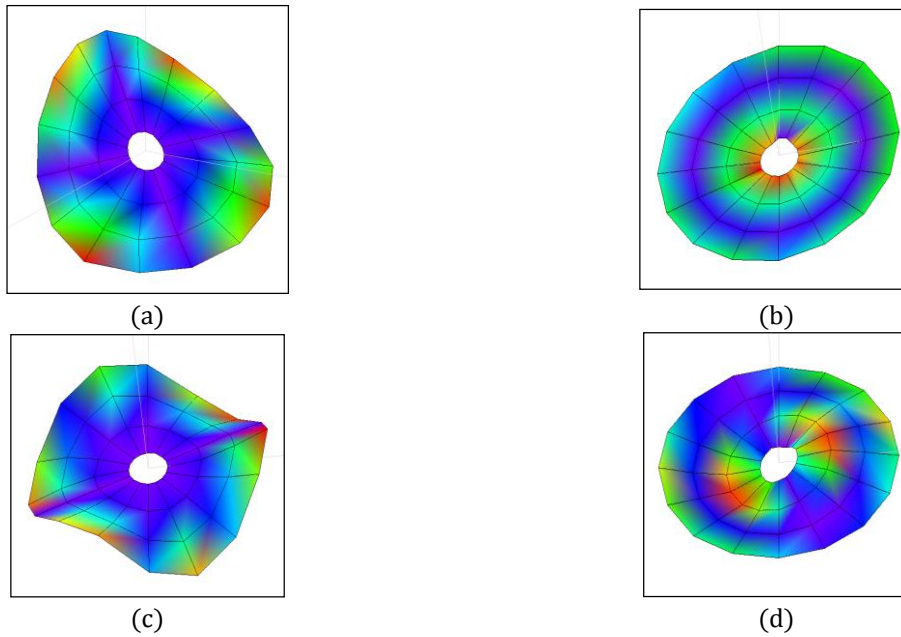
The first six natural frequencies of annular plate are measured and listed in Table 1, and compared with those obtained from the numerical simulation. As would be observed, the natural frequencies obtained from the experiment are in good agreement with those obtained from the simulation. Figure 3 shows the FRF curves at several measured points. FRF curves are given in terms of the point acceleration versus the natural frequency. Figure 4 illustrates the first four mode shapes obtained from experimental modal analysis. Variation of the first four natural frequencies of an annular plate is exhibited in Figure 5 for two different values of the aspect ratio (inner radius/outer radius) of the plate. Aspect ratio of 0.25 relates to a plate with inner radius of the hole 20 mm and outer radius of 80 mm, while, the value of 0.16 relates to the inner radius of 20 mm and the outer radius of 125 mm. The results are also compared with those of a circular plate without hole. As can be seen, the natural frequencies of the annular plate increase as the aspect ratio increases. This is mainly due to the fact that as the aspect ratio increases, the effective mass of the structure decreases and the natural frequency increases as a consequence.

**Table 1.** The first six natural frequencies (Hz) of vibration for a circular plate with central hole

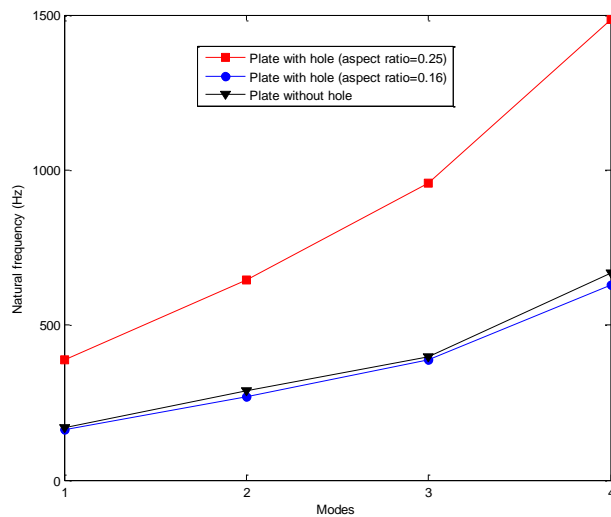
Mode number	Experiment	Numeric	Error (%)
1	168.78	163.68	3.02
2	280.97	268.84	4.31
3	399.64	389.47	2.54
4	633.83	629.47	0.68
5	705.59	683.92	3.07
6	982.24	1048.2	6.71



**Figure 3.** FRF curves at different measured points



**Figure 4.** First four mode shapes obtained from experimental modal analysis



**Figure 5.** Comparison of natural frequency for different aspect ratios

## 5. Conclusions

The aim of the present investigation is towards the understanding of the vibrational behavior of circular plates with central hole. Experimental modal analysis is performed in order to achieve the modal parameters of the plate through the FRF curves at several points on the plate. A three-dimensional finite element of the plate is also constructed in Abaqus 6.10 software. The results of the two analyses meet an excellent agreement. Results show that as the aspect ratio increases the natural frequency increases. This is because of the fact that as the aspect ratio increases, the effective mass of the structure decreases and the natural frequency increases as a consequence.

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